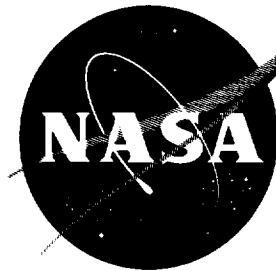


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TECHNICAL NOTE

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A METHOD FOR PREDICTING THE STATIC STRENGTH OF A
STIFFENED SHEET CONTAINING A SHARP CENTRAL NOTCH

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SUMMARY

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A method for predicting the residual static strength of a stiffened sheet with a central notch is presented. The structural configurations treated consist of a thin sheet with a sharp notch and a stiffener centrally located over the notch. Residual static strengths are predicted for three structural configurations having ratios of notch area to stiffener area equal to 0.2, 1.0, and 5.0. Two aluminum alloys, 2024-T3 and 7075-T6, were investigated. The predicted results are compared with a limited number of experimental results and fair agreement is obtained.

INTRODUCTION

The fail-safe design philosophy requires that the designer have a basic knowledge of the residual static strength of structural parts containing cracks. The National Aeronautics and Space Administration has conducted several investigations pertaining to residual static strength (refs. 1, 2, and 3) which present the results of static-strength tests of simple and built-up structures containing fatigue cracks and a simple method for predicting their residual static strength. It was found from these investigations that it becomes increasingly more difficult to predict the residual strength as the structure becomes more complex. This difficulty is partly due to the complicated manner in which the load is distributed throughout a complex structure.

The present investigation is intended to extend the method of reference 3 to include simple built-up structures which consist of a thin sheet containing a sharp notch and a stiffener centrally located over the notch. The method utilizes the approach of reference 3 and incorporates the coefficients derived in reference 4. These coefficients were obtained analytically and are used to determine the effect of a reinforcing stiffener on the stress concentration factor at the tip of a crack in a thin sheet. In order to evaluate the method, a limited number of tests were conducted on several structural configurations constructed of both 2024-T3 and 7075-T6 aluminum alloys.

SYMBOLS

| | |
|-----------|--|
| A_c | notch area (notch length times sheet thickness), sq in. |
| A_n | net area of cross section containing notch (sheet and stiffener), sq in. |
| A_s | stiffener area, sq in. |
| C | coefficient to modify for effects of reinforcing stiffener on stress concentration factor in sheet |
| D | rivet diameter, in. |
| E | Young's modulus, ksi |
| $E_{s,n}$ | secant modulus for nominal net stress at notched cross section, ksi |
| $E_{s,u}$ | secant modulus corresponding to stress at ultimate load, ksi |
| K | stress concentration factor |
| K_N | Neuber technical stress concentration factor |
| K_T | theoretical stress concentration factor |
| K_T' | theoretical stress concentration factor modified for effect of reinforcing stiffener |
| K_u | stress concentration factor for ultimate tensile strength |
| K_w | factor providing for finite width of sheet |
| l | notch length, in. |
| p | rivet pitch, in. |
| P | load, lb |
| P_e | experimental maximum load, lb |
| P_p | predicted maximum load, lb |
| S_n | nominal net section stress, ksi |
| t_s | thickness of stiffener, in. |

| | |
|------------|---|
| W | width of sheet, in. |
| W_s | width of stiffener, in. |
| λ | ratio of notch area A_c to stiffener area A_s |
| σ | local stress at tip of notch, ksi |
| σ_u | ultimate tensile strength, ksi |
| σ_y | yield strength, 0.2-percent offset, ksi |
| ρ | radius at tip of notch, in. |
| ρ' | material constant |

METHOD

A simple method for determining the residual static strength of sheet specimens containing a crack has been developed by McEvily, Illg, and Hardrath (ref. 1), and improved by Kuhn and Fligge (ref. 3). The method is based on the determination of the maximum stress which occurs at the tip of the crack in the sheet. The residual static strength (hereafter called residual strength or strength) of the sheet is determined by assuming that failure occurs whenever the maximum local stress equals the ultimate tensile strength of the material. Sanders (ref. 4) has determined analytically that a stiffener, attached to a sheet containing a crack, and centrally located over the crack, significantly reduces the effect of the stress concentration at the tip of the crack. It is shown in reference 4 that the reduction in stress concentration at the tip of the crack due to the stiffener is a function of the ratio of crack area to stiffener area. The results of these investigations are utilized to develop an expression for predicting residual static strength of stiffened panels. Although, in reference 4, a stress concentration factor for the stiffener is also determined, this factor was not incorporated into the method developed in the present paper because of difficulties in accounting for the effects of plastic deformations in the stiffener.

The configuration to be analyzed consists of a thin sheet containing a sharp internal notch with a stiffener centrally located over the notch. (See fig. 1.) From elementary theory, the load on the specimen is the product of the net area and the nominal net section stress. The local stress at the tip of the notch σ is the product of the nominal net section stress S_n and a stress concentration factor K . It is assumed, as in references 1 and 3, that at failure $\sigma = \sigma_u$. Thus

$$\sigma = \sigma_u = S_n K_u \quad (1)$$

where K_u is the stress concentration factor at failure discussed in reference 3.

It is also assumed that the nominal net section stress in the stiffener and in the sheet are equal at failure. It follows then that the failing load is equal to the product of the net area of the sheet and stiffener and the nominal net section stress in the sheet at failure. Thus,

$$P_p = A_n \frac{\sigma_u}{K_u} \quad (2)$$

The only unknown on the right-hand side of this equation is the factor of stress concentration K_u . The following paragraphs outline a method for calculating K_u based on the results of references 3 and 4.

A theoretical stress concentration factor K_T is calculated and subsequently corrected for size and plasticity effects. The factor K_T was calculated by using the following equation proposed by Kuhn (ref. 5):

$$K_T = 1 + 2K_w \sqrt{\frac{l}{2\rho}} \quad (3)$$

The parameter K_w is an empirically adjusted factor to correct for finite width, based on photoelastic results obtained by Dixon (ref. 6), and is a function of l/W . The parameter K_w is expressed mathematically as follows:

$$K_w = \sqrt{\frac{1 - \frac{l}{W}}{1 + \frac{l}{W}}} \quad (4)$$

The factor K_T must now be modified to account for the presence of the stiffener. A new theoretical stress concentration factor K_T' of the sheet-stiffener combination is obtained by taking the product of Sanders' factor C (ref. 4) and the factor K_T . Thus

$$K_T' = CK_T \quad (5)$$

The theoretical stress concentration factor K_T' is then corrected for material size effects by using the following equation (ref. 3):

$$K_N = 1 + \frac{K_T' - 1}{1 + \sqrt{\frac{\rho'}{\rho}}} \quad (6)$$

Finally, a plasticity correction is made with the following equation which results in the desired stress concentration factor K_u (ref. 3):

$$K_u = 1 + (K_N - 1) \frac{E_{s,u}}{E_{s,n}} \quad (7)$$

The quantity K_u is the stress concentration factor in the sheet at failure which incorporates the corrections for size and plasticity effects and also the effect of a reinforcing stiffener. At this point all the required parameters have been determined for making a prediction of the residual strength of a sheet-stiffener combination.

The preceding equations have been used to predict the residual strengths of three structural configurations having ratios of notch area to stiffener area λ equal to 0.2, 1.0, and 5.0. The two extreme values of λ were considered to be the practical limits encountered in structural parts. The material constant ρ' and the factor C used in the analysis were obtained from plots in references 3 and 4, respectively. For convenience, these plots have been reproduced in figures 2 and 3. Material properties used for computing K_u are listed in table I. For convenience, a plot of K_u against K_N for the materials tested is presented in figure 4. The predicted results are compared with a limited number of test results in subsequent sections.

EXPERIMENTS

Specimens

Specimens were constructed of both 2024-T3 and 7075-T6 aluminum-alloy (hereafter designated 2024 and 7075) sheet having nominal dimensions of 12 by 36 by 0.064 inches (fig. 5). In total, 24 specimens were tested. In most cases the stiffener was cut from the same stock as the sheet material and riveted to the sheet with 2117 aluminum-alloy rivets. In most cases the stiffeners were riveted to one side of the sheet only. In cases where large stiffener areas were required, two stiffeners were used, one being riveted to each side. In two cases an angle stiffener was riveted to one side of the sheet. The distinguishing features of each specimen are listed in table II.

The specimens were constructed so that they would have the same values of λ for which residual strengths were predicted; that is, 0.2, 1.0, and 5.0. The notch was made by drilling a 1/16-inch-diameter hole in the center of the sheet and then inserting a jeweler's saw into the hole to cut the notch. The last 1/32 inch at each end of the notch was made by drawing a thread, impregnated with valve grinding compound, over the edge to be cut with a reciprocating motion. This procedure gave a radius of curvature equal to 0.0050 ± 0.001 inch at the tip of the notch. The rivet pitch used in attaching the stiffener to the sheet was three times the rivet diameter except for two cases as indicated in table II. Specimens having a value of $\lambda = 0.2$ were constructed with 7075 material only because of a lack of 2024 material of the size required for the stiffeners. The

7075 specimens having a value of λ equal to 0.2 were made with the stiffener extended up into the grip to help distribute the load between the sheet and stiffener and prevent failure at the grip line. In one case the stiffener was riveted to the sheet between the grips and bonded to the sheet in the grips. In another case the stiffener was riveted to the sheet between the grips and bonded to the sheet in and between the grips (full length).

The tensile properties of the materials used were obtained by averaging the results of 6 tests on each material. The average tensile properties are listed in table I and typical stress-strain curves are presented in figures 6 and 7. Tests were conducted on standard ASTM tensile specimens in a 120,000-pound-capacity universal static testing machine. A 0.001-inch-division dial gage was used to record the strains over a 2-inch-gage length. Load and strain were recorded simultaneously.

Test Equipment and Procedure

The tests were conducted in a 120,000-pound-capacity universal hydraulic testing machine which is described in reference 7. The specimen grips were pin-connected at the top and fixed at the bottom of the machine. Five 1-inch bolts were used to clamp the specimen in each grip. Plastic liners were placed between the specimen and the grips to produce a reasonably uniform pressure over the entire clamping area. Once clamped the specimens were subjected to a uniformly increasing tension load until failure occurred.

COMPARISON BETWEEN PREDICTED AND EXPERIMENTAL RESULTS

The method described in this report was used to predict the residual static strength of the stiffened sheets. The predicted results are presented as curves in figures 8 to 12. Also plotted in figures 8 to 12 are the results of the tests on stiffened sheets which were conducted in order to evaluate the method.

It should be noted that in the prediction it is assumed that the sheet fails first and results in failure of the entire specimen. In conducting the tests it could not usually be distinguished which failed first, the stiffener or the sheet. The predicted and experimental results for the configurations and materials investigated are discussed in subsequent paragraphs.

2024 Aluminum-Alloy Specimens

$\lambda = 1.0$. - For the case where the stiffener area was equal to the notch area, the stiffener was very effective in reducing the stress concentration in the sheet. The strengths of the specimens tested are approximately equal regardless of the notch length used; thus, the stress concentration in the sheet was essentially independent of notch length. Good agreement between predicted and experimental results was obtained. (See fig. 8.)

One specimen in this group was tested with an angle stiffener rather than with a strap stiffener and no significant difference in strength was noted. (See fig. 8.) However, another investigation has found (ref. 8) that the strength is a function of the percentage of stiffener area in contact with the sheet, that is, the greater the contact area the higher the strength.

$\lambda = 5.0$.— The experimental results show progressively lower strengths for longer notch lengths (fig. 9) and are in good agreement with the predicted results. In this case the stiffener was not as effective in reducing the stress concentration in the sheet as were 2024 specimens with $\lambda = 1.0$. This result is as expected, since the factor C approaches unity as the stiffener size is reduced.

7075 Aluminum-Alloy Specimens

$\lambda = 0.2$.— These specimens were constructed so that the stiffener extended up into the grips in order to prevent failure at the grip line. One specimen had a $1\frac{1}{2}$ -inch notch length and had the stiffener riveted to the sheet between the grips and bonded to the sheet in the grips. The other specimen had a 4-inch notch length and had the stiffener bonded and riveted the full length between grips. The fact that the specimen tested with the 4-inch notch had a higher strength than the specimen having a $1\frac{1}{2}$ -inch notch can be attributed to the much larger stiffener employed in the specimen with the 4-inch notch and not to the difference in fabrication techniques. It was expected that the sheet would fail first because of the large stiffeners employed. However, as mentioned previously, it could not be distinguished which failed first, the sheet or the stiffener. Fair agreement between predicted and actual strengths was obtained. (See fig. 10.)

$\lambda = 1.0$.— It appears that the stiffeners were not as effective in reducing the effect of the stress concentration in the 7075 sheet as they were for similar 2024 specimens ($\lambda = 1.0$). Two additional parameters were investigated in this group of specimens. First, the effect of strap and angle stiffeners was investigated. The results (fig. 11) indicate a higher strength for the specimen with the strap stiffener; this result is in agreement with the results reported in reference 8. A second parameter investigated was rivet pitch. These results are also presented in figure 11. Progressively lower strengths were obtained as the rivet pitch was increased up to $5\frac{1}{2}$ inches. In the specimen with a $2\frac{3}{4}$ -inch rivet pitch ($D = \frac{3}{16}$ inch), the sheet failed; then the stiffener failed. In the specimen with a $5\frac{1}{2}$ -inch rivet pitch ($D = \frac{3}{16}$ inch), the sheet failed and then the rivets failed but the stiffener did not fail. Progressively larger rivet pitches reduce the effect the stiffener has on the stress concentration in the sheet. Experimental results for this series of specimen configurations were in fair agreement with the predicted results except for the cases of large rivet pitch and angle stiffener. (See fig. 11.) Tests that were conducted to determine the effects of

rivet pitch and stiffener shape were not expected to be in agreement with the predicted results. The results of these tests do indicate that the effectiveness of stiffeners is reduced as rivet pitch is increased.

$\lambda = 5.0$. - The experimental results were approximately the same as the experimental results for 2024 specimens of the same configuration, that is, progressively lower strengths for longer notch lengths. (See fig. 12.) Fair agreement between predicted and actual strengths was obtained. The stiffener was not as effective in reducing the stress concentration in the sheet as for 7075 specimens with $\lambda = 1.0$. As in the corresponding case for 2024 specimens with $\lambda = 5.0$, this result can be explained by the fact that the coefficient C approaches unity as the stiffener size is reduced.

CONCLUDING REMARKS

A method for predicting the residual static strength of a stiffened sheet containing a central notch is presented. The method consists of determining the load required to produce the ultimate tensile strength of the material in the sheet at the tip of the notch. It is assumed that the sheet fails first and causes failure of the entire structure. The predicted stress in the sheet has been modified to take into account the effect the reinforcing stiffener has on the stress concentration in the sheet and is corrected for material size effects and plasticity. It should be noted that there is a stress concentration associated with the stiffener which has not been accounted for in the method presented. For the configuration treated, this omission does not appear to be important; however, it might be important in treating other configurations.

Tests were performed on specimens of 2024-T3 and 7075-T6 aluminum alloys in order to evaluate the method. Fair agreement between predicted and experimental results was obtained for the two materials investigated. Additional tests revealed that a large increase in rivet pitch resulted in a decrease in strength and also that strap stiffeners had slightly higher strengths than angle stiffeners of the same cross-sectional area.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., May 21, 1963.

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TABLE I.- AVERAGE TENSILE PROPERTIES OF SHEET AND STIFFENER MATERIAL

| Material | Yield strength, 0.2-percent offset, ksi | Ultimate tensile strength, ksi | Total elongation, percent 2-in.-gage length | Young's modulus, ksi |
|----------|---|--------------------------------------|---|-------------------------|
| 2024-T4 | 51.2 | 72.2 | 19.9 | 10.7×10^3 |
| 7075-T6 | 75.4 | 82.4 | 11.8 | 10.4 |

TABLE II.- SPECIMEN GEOMETRY AND TEST RESULTS

[36 by 12 by 0.064 in. stiffened sheet containing a central notch with a radius of 0.005 in.]

| Specimen cross section | Notch length, l, in. | Notch area, A _c , sq in. | Stiffener width, W _s , in. | Stiffener thickness, t _s , in. | Stiffener area, A _s , sq in. | λ | Rivet diameter, D, in. | Rivet pitch, P, in. | Experimental load, P _e , kips |
|------------------------|----------------------|-------------------------------------|---------------------------------------|---|---|-----|------------------------|---------------------|--|
| 2024-T3 aluminum alloy | | | | | | | | | |
| | 0.50 | 0.032 | 0.50 | 0.064 | 0.032 | 1.0 | 1/8 | 3/8 | 32.6 |
| | 1.50 | 0.096 | 2 at 0.75 or 1.50 | 0.064 | 0.096 | 1.0 | 1/8 | 3/8 | 40.0 |
| | 2.78 | 0.178 | 1 x 1 angle | 3/32 | 0.178 | 1.0 | 3/16 | 9/16 | 138.8 |
| | 2.78 | 0.178 | 2 at 2.0 or 4.0 | 0.064 | 0.178 | 1.0 | 3/16 | 9/16 | 39.4 |
| | 4.00 | 0.256 | 2 at 3.0 or 6.0 | 0.064 | 0.256 | 1.0 | 3/16 | 9/16 | 38.8 |
| | 6.00 | 0.384 | 2 at 3.0 or 6.0 | 0.064 | 0.384 | 1.0 | 3/16 | 9/16 | 37.8 |
| | 1.00 | 0.064 | 0.51 | 0.025 | 0.0128 | 5.0 | 1/8 | 3/8 | 37.9 |
| | 2.50 | 0.160 | 0.50 | 0.064 | 0.032 | 5.0 | 1/8 | 3/8 | 32.3 |
| | 4.00 | 0.256 | 0.80 | 0.064 | 0.0512 | 5.0 | 1/8 | 3/8 | 26.6 |
| | 6.00 | 0.384 | 1.20 | 0.064 | 0.0768 | 5.0 | 3/16 | 9/16 | 20.8 |
| 7075-T6 aluminum alloy | | | | | | | | | |
| | 1.50 | 0.096 | 2 at 3.75 or 7.5 | 0.064 | 0.096 | 0.2 | 3/16 | 9/16 | 270.6 |
| | 4.00 | 0.256 | 2 at 2.56 or 5.12 | 0.250 | 1.280 | 0.2 | 3/16 | 9/16 | 398.7 |
| | 0.50 | 0.032 | 0.50 | 0.064 | 0.032 | 1.0 | 1/8 | 3/8 | 51.2 |
| | 1.50 | 0.096 | 2 at 0.75 or 1.50 | 0.064 | 0.096 | 1.0 | 1/8 | 3/8 | 43.0 |
| | 2.78 | 0.178 | 1 x 1 angle | 3/32 | 0.178 | 1.0 | 3/16 | 9/16 | 133.0 |
| | 2.78 | 0.178 | 2.78 | 0.064 | 0.178 | 1.0 | 3/16 | 9/16 | 37.4 |
| | 2.78 | 0.178 | 2.78 | 0.064 | 0.178 | 1.0 | 3/16 | 2 3/4 | 431.0 |
| | 2.78 | 0.178 | 2.78 | 0.064 | 0.178 | 1.0 | 3/16 | 5 1/2 | 528.6 |
| | 6.00 | 0.384 | 2 at 3.0 or 6.0 | 0.064 | 0.384 | 1.0 | 3/16 | 9/16 | 32.0 |
| | 6.00 | 0.384 | 2 at 3.0 or 6.0 | 0.064 | 0.384 | 1.0 | 3/16 | 9/16 | 34.2 |
| | 1.00 | 0.064 | 0.51 | 0.025 | 0.0128 | 5.0 | 1/8 | 3/8 | 40.0 |
| | 2.50 | 0.160 | 0.50 | 0.064 | 0.032 | 5.0 | 1/8 | 3/8 | 27.4 |
| | 4.00 | 0.256 | 0.80 | 0.064 | 0.0512 | 5.0 | 1/8 | 3/8 | 23.2 |
| | 6.00 | 0.384 | 1.20 | 0.064 | 0.0768 | 5.0 | 3/16 | 9/16 | 18.9 |

¹Angle stiffener.

²Stiffener bonded in grips only.

³Stiffener bonded full length.

⁴Sheet failed first, then stiffener.

⁵Sheet failed first, then rivets.

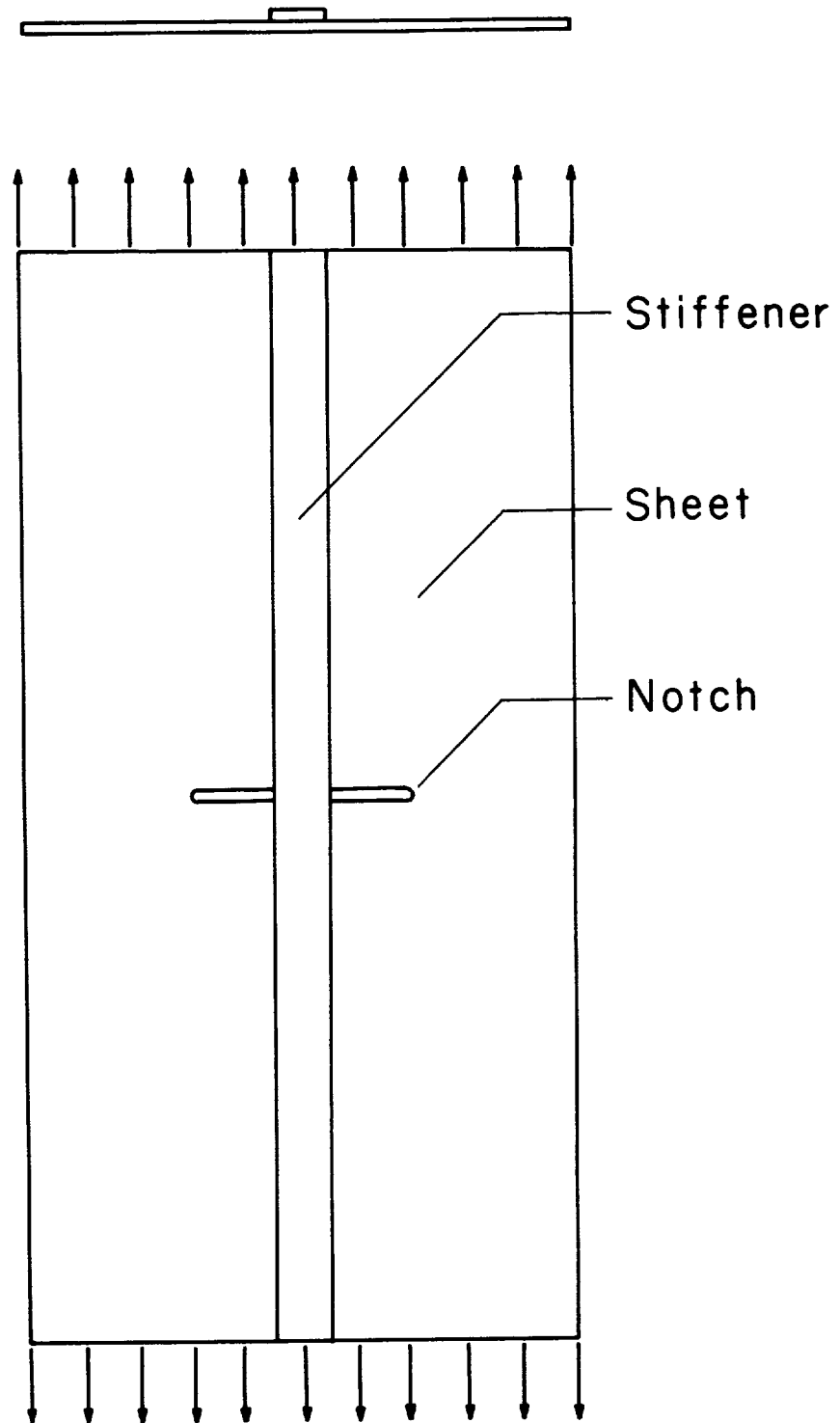


Figure 1.- Sheet-stiffener configuration.

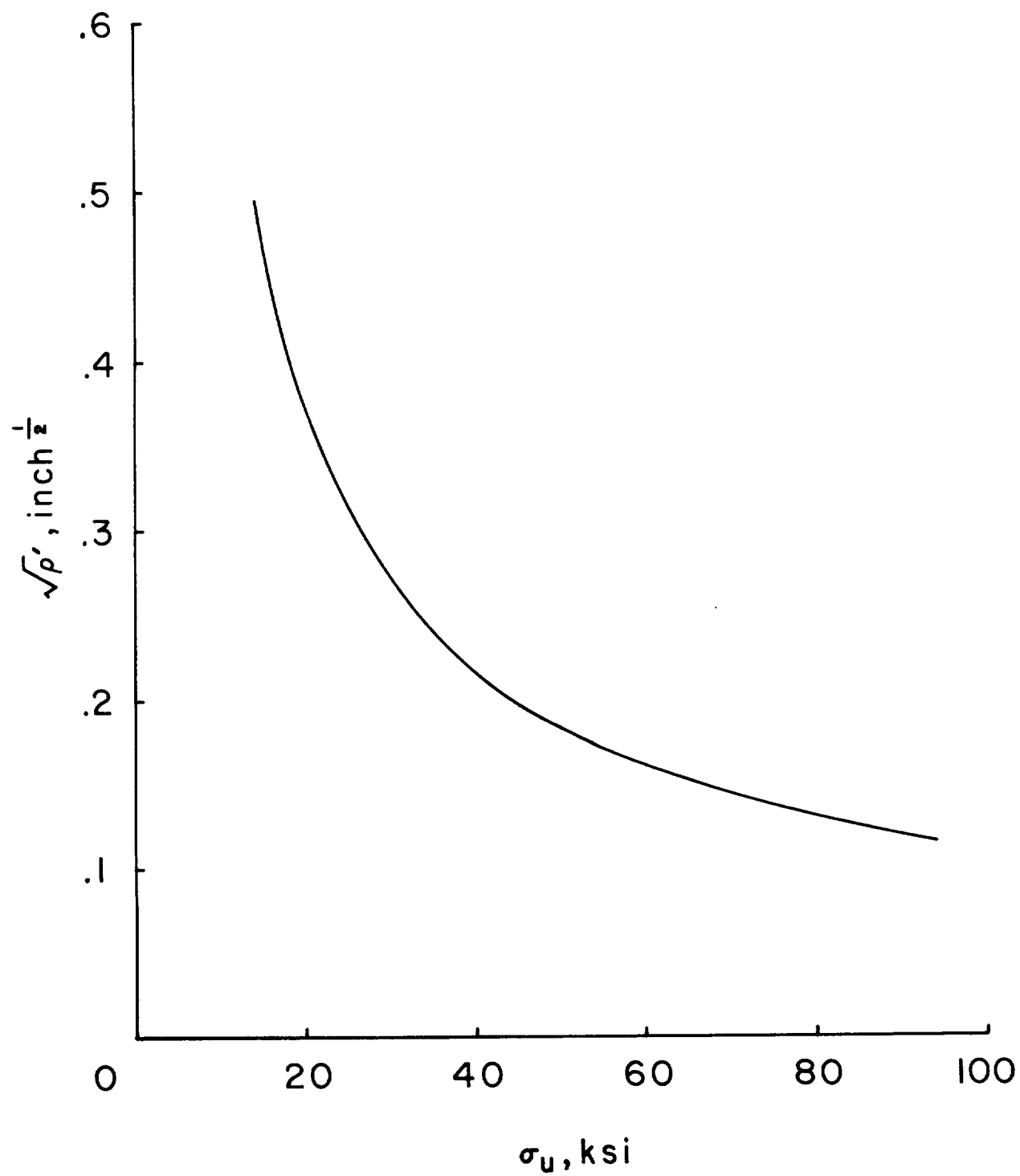


Figure 2.- Relation between $\sqrt{\dot{\rho}'}$ and σ_u for wrought aluminum alloys in the heat-treated condition. (From ref. 3.)

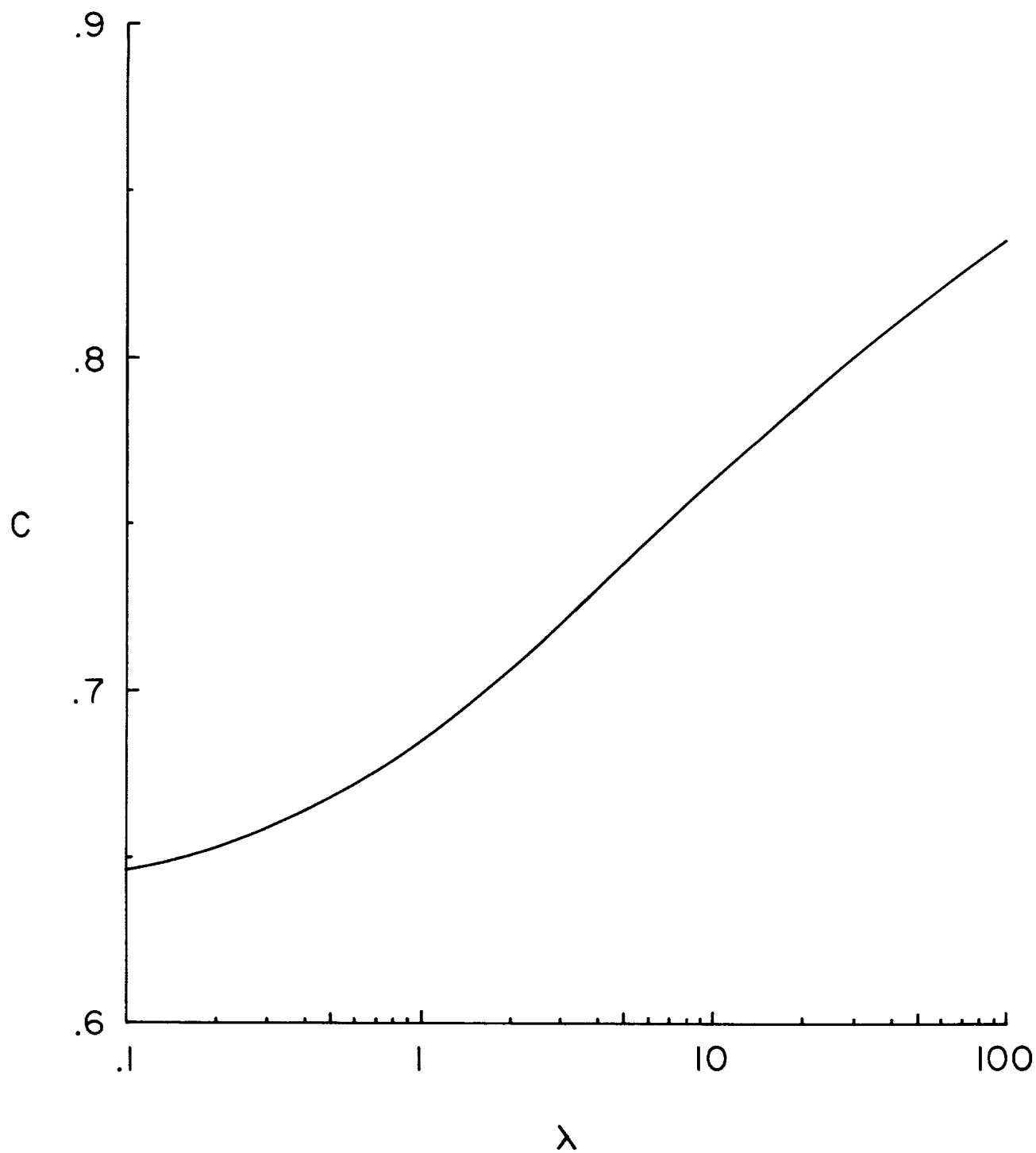


Figure 3.- Variation of the coefficient C with the ratio λ of notch area to stiffener area.
(From ref. 4.)

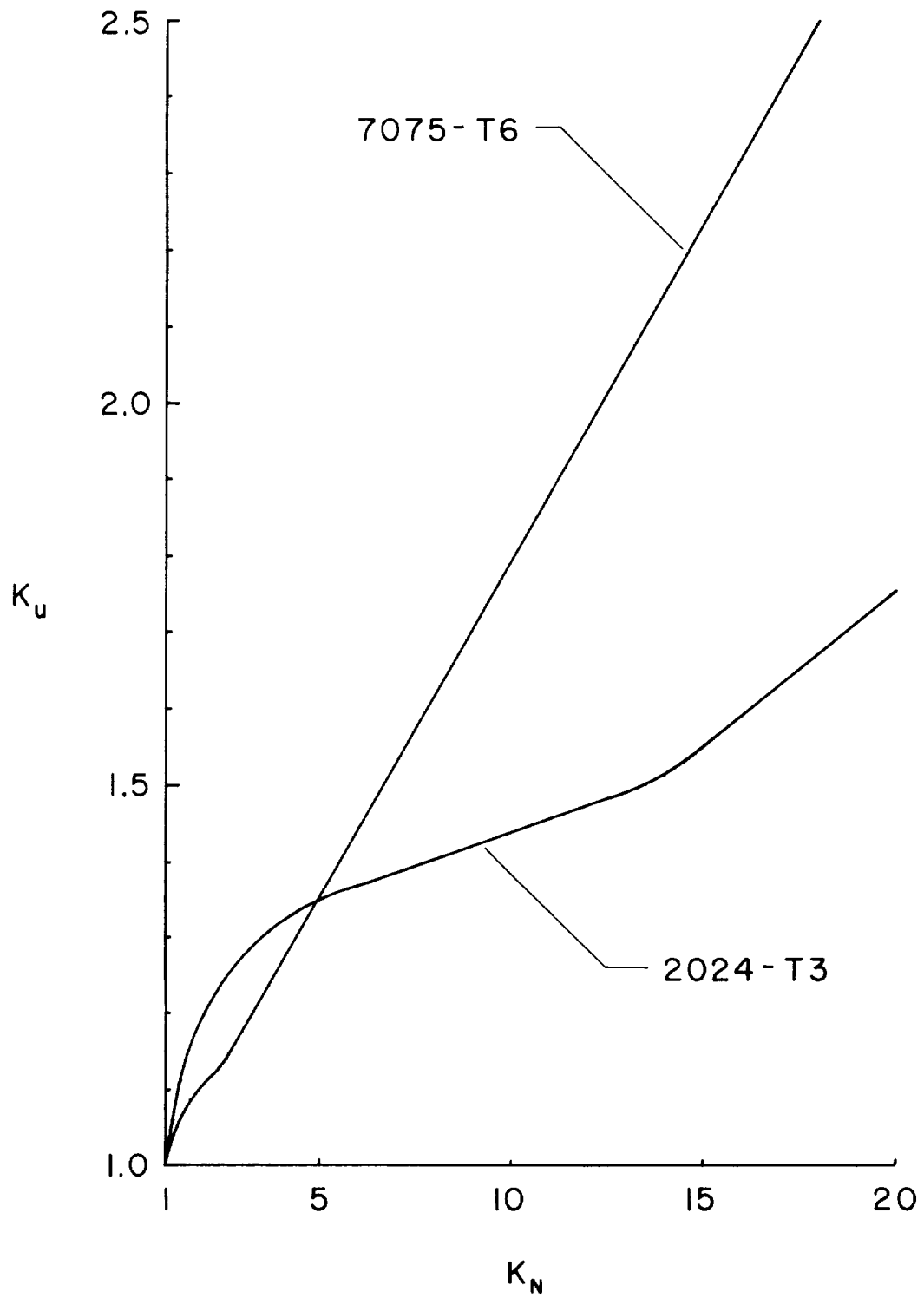


Figure 4.- Relation between K_u and K_N .

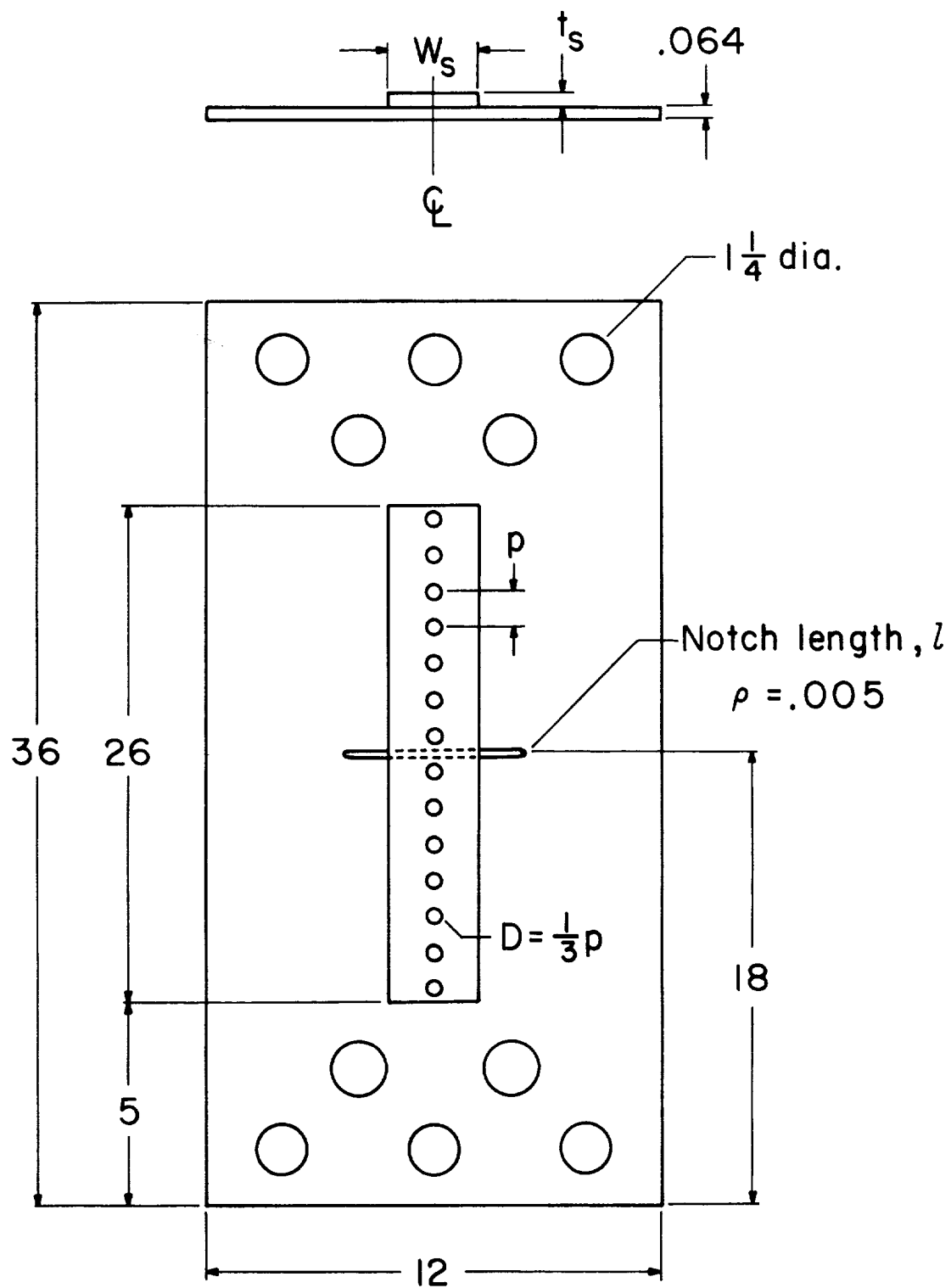


Figure 5.- Typical specimen configuration. All dimensions are in inches.

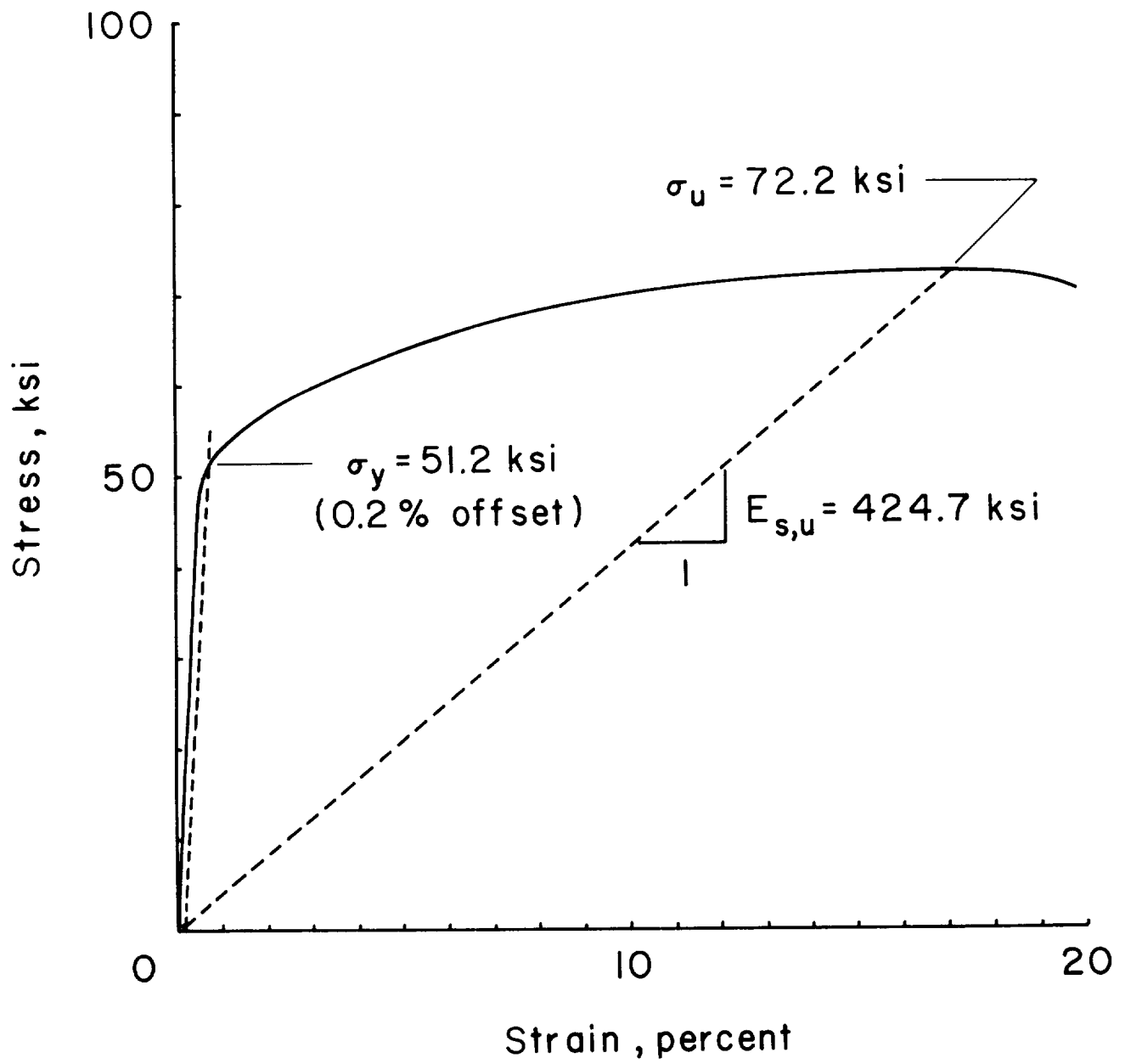


Figure 6.- Typical stress-strain curve for 2024-T3 aluminum-alloy sheet. $E = 10,700 \text{ ksi}$.

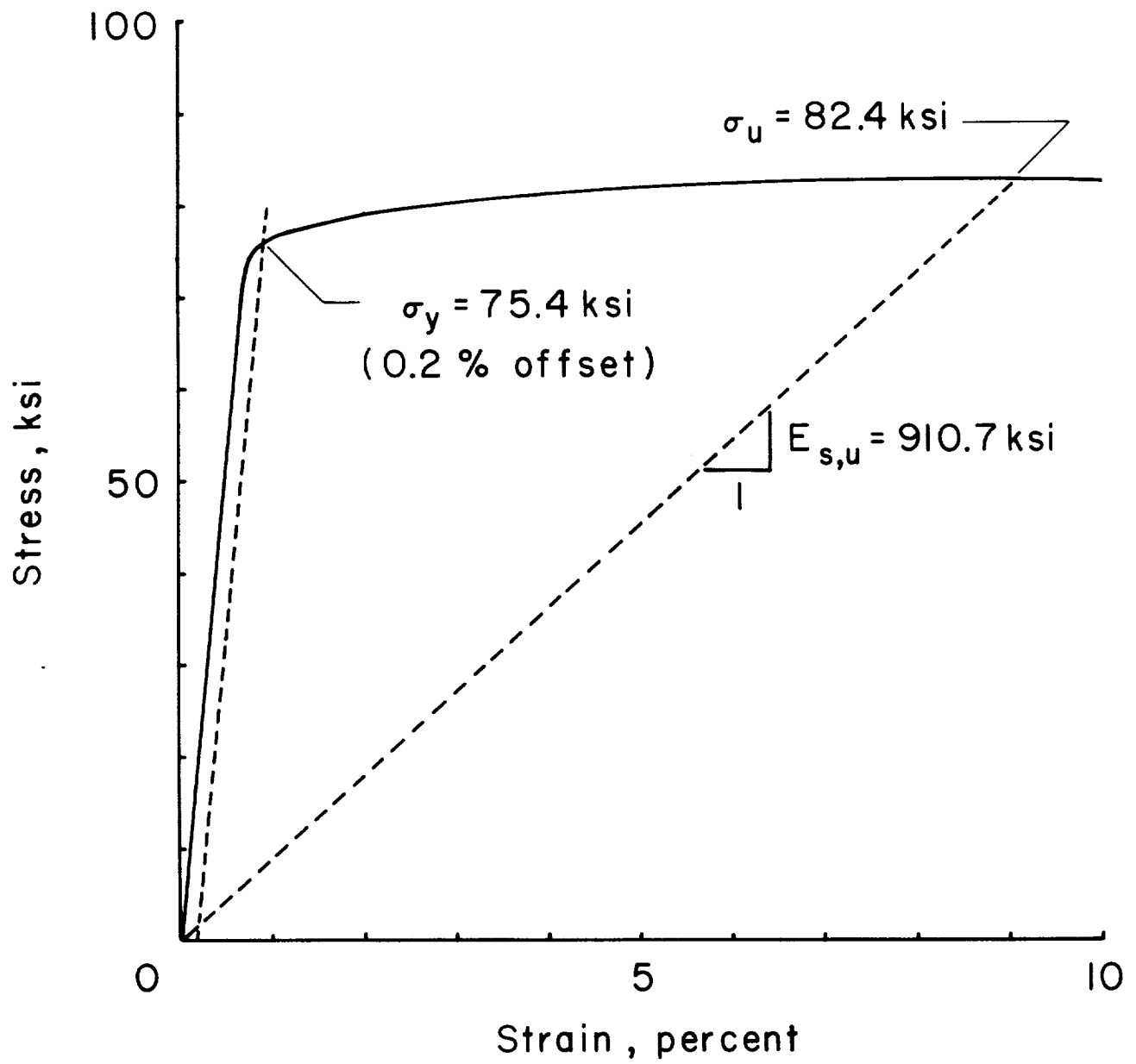


Figure 7.- Typical stress-strain curve for 7075-T6 aluminum-alloy sheet. $E = 10,400$ ksi.

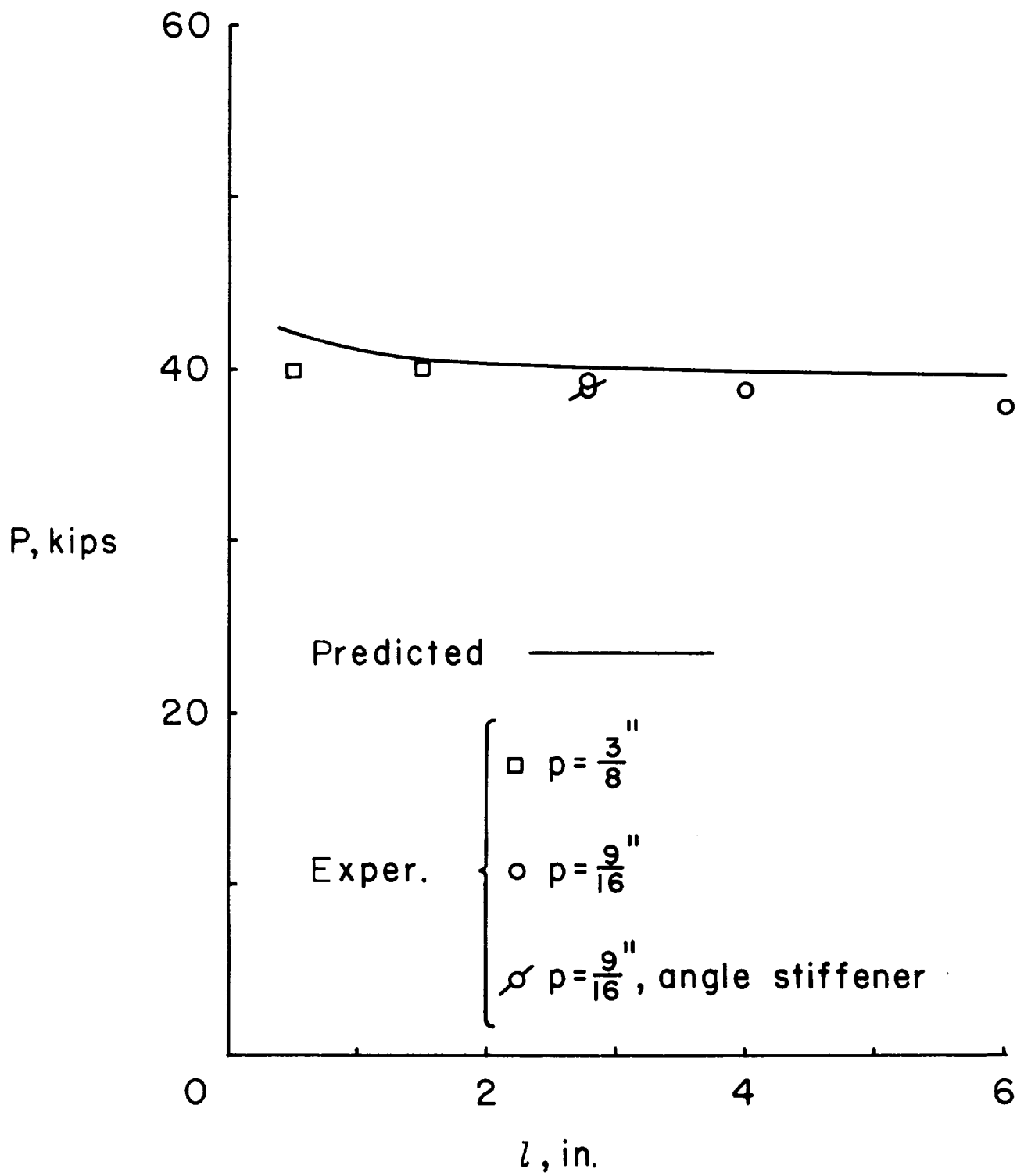


Figure 8.- Experimental and predicted residual strengths of 2024-T3 aluminum-alloy specimens.
 $\lambda = 1.0$.

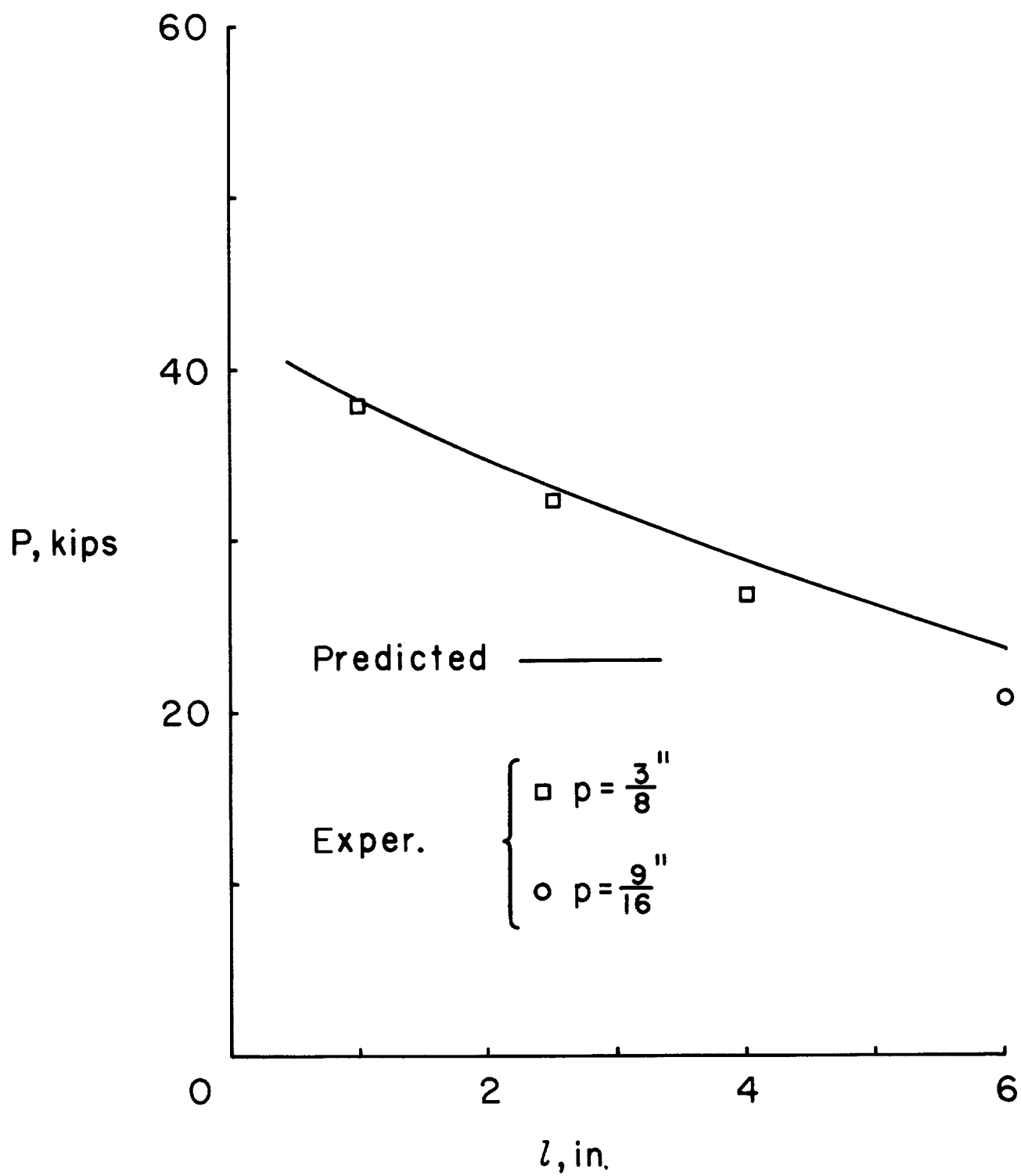


Figure 9.- Experimental and predicted residual strengths of 2024-T3 aluminum-alloy specimens.
 $\lambda = 5.0$.

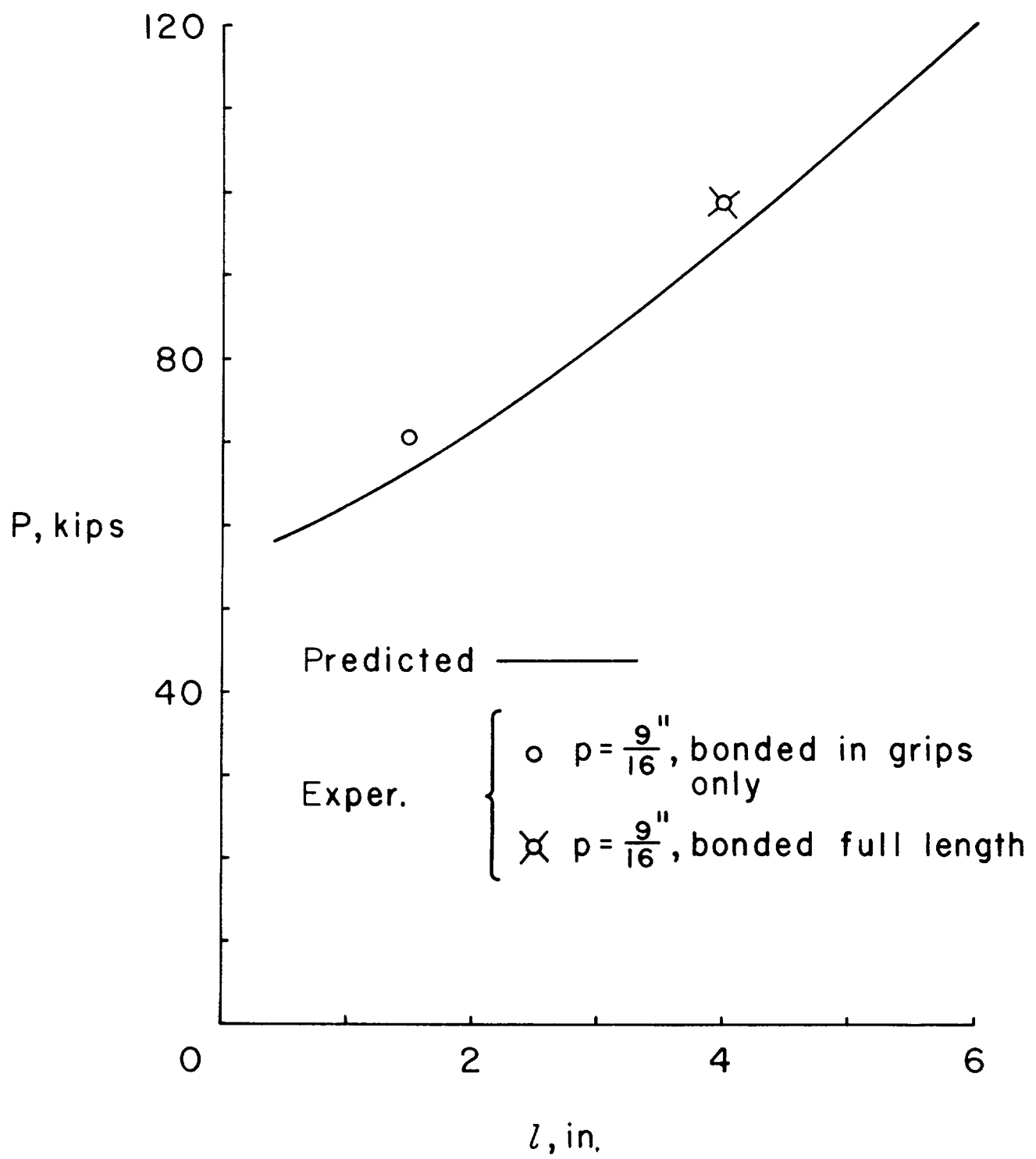


Figure 10.- Experimental and predicted residual strengths of 7075-T6 aluminum-alloy specimens.
 $\lambda = 0.2$.

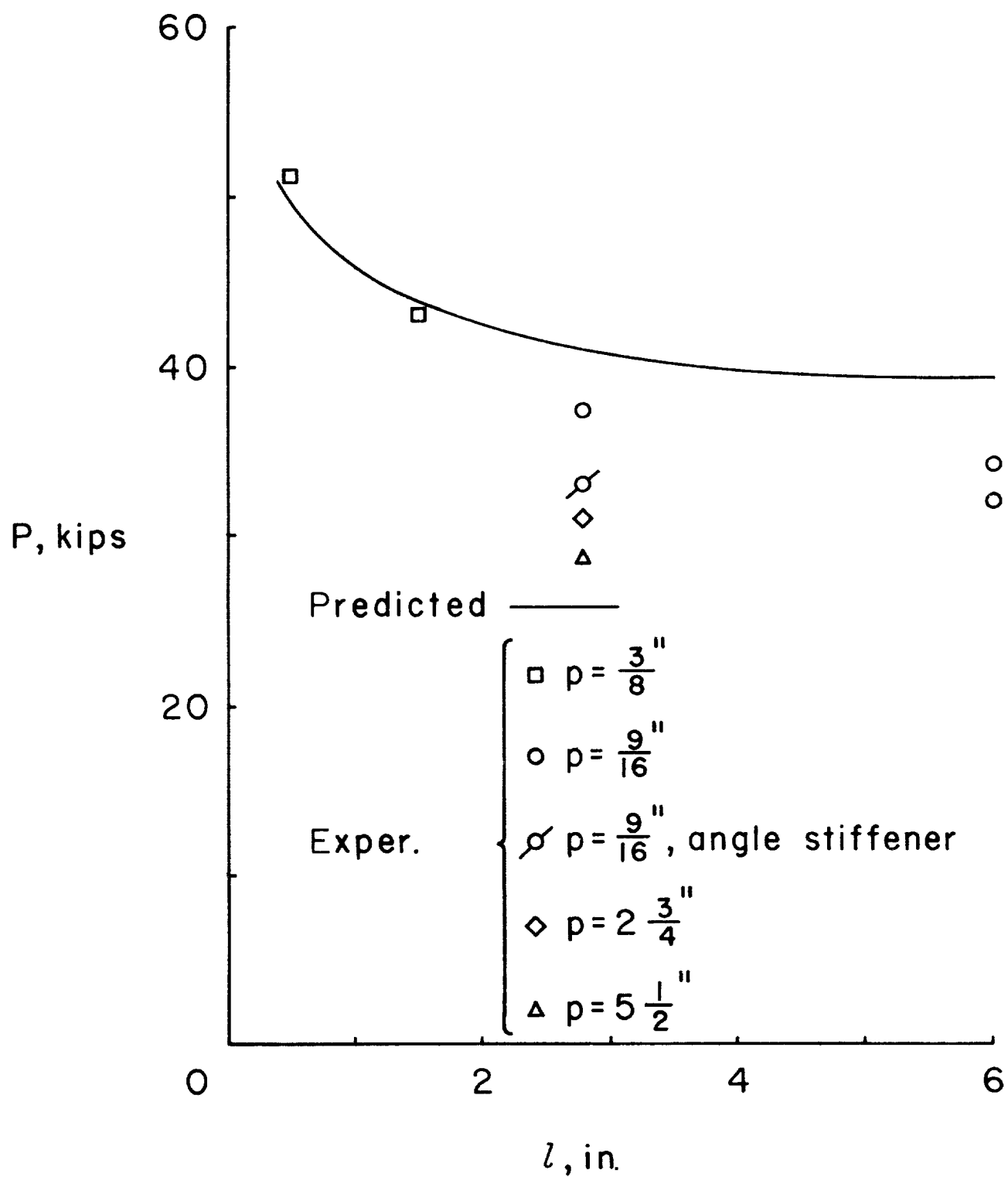


Figure 11.- Experimental and predicted residual strengths of 7075-T6 aluminum-alloy specimens.
 $\lambda = 1.0$.

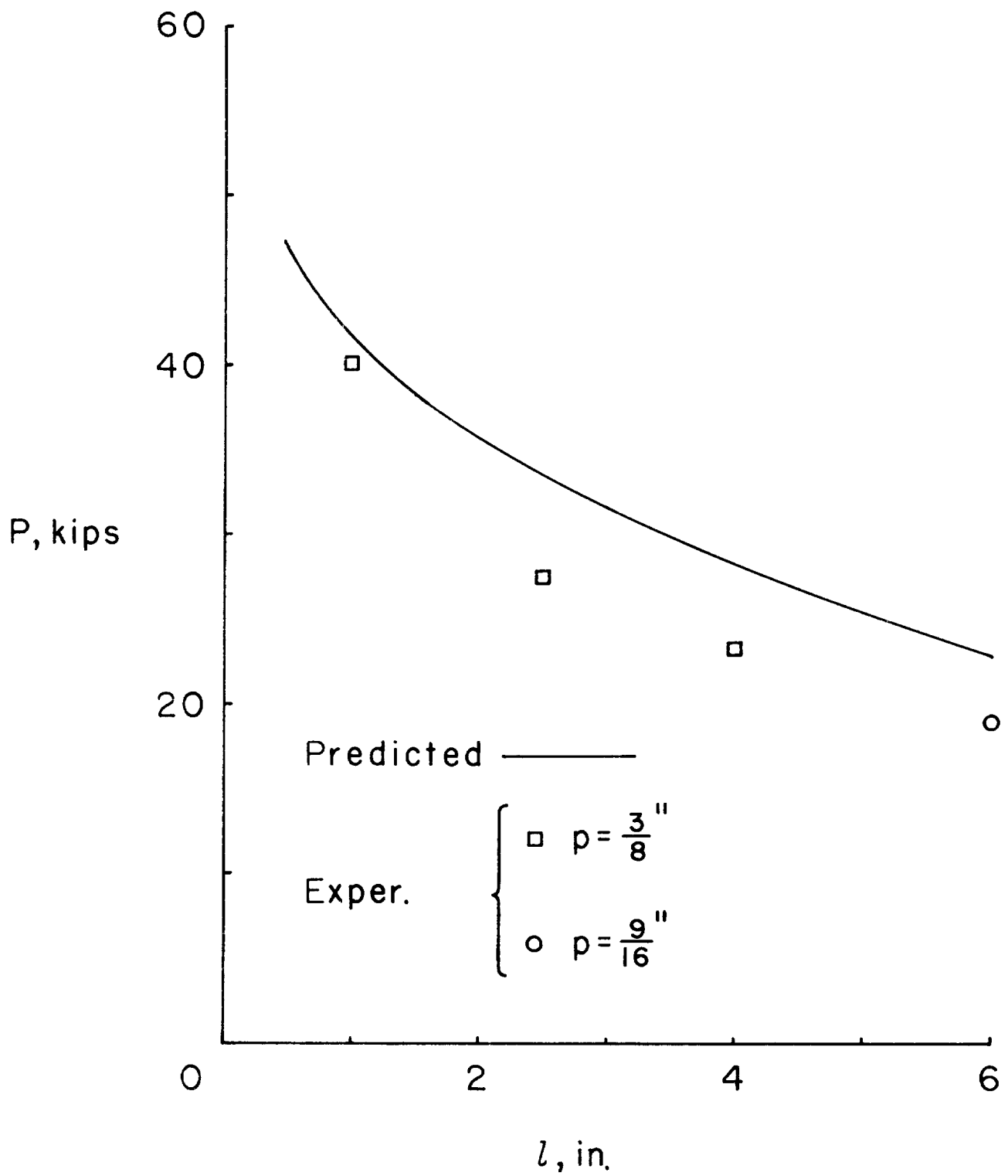


Figure 1.- Experimental and predicted residual strengths of VOYD-TM aluminum alloy specimens.
 $\lambda = 5.0$.

